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A TEST OF SUEHRCKE'S SUNSHINE–RADIATION RELATIONSHIP USING A GLOBAL DATA SET

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Abstract—In this paper we investigate a new relationship between duration of sunshine and solar radiation on the earth's surface that was derived recently by Suehrcke (Solar Energy, 68(5) (2000) 417). We test the relationship using over 70,000 measured monthly sunshine and radiation data from nearly 700 sites compiled by the World Radiation Data Center. We show that Suehrcke's equation accounts adequately for the sunshine–radiation relationship on an average sense. There is a large dispersion (12% on average) in the values of solar radiation calculated by the new equation, however it is unclear how much of this dispersion could be accounted for by a better model. The predictive capabilities of the new model are actually roughly equivalent to those of older models such as Ångström–Prescott when the peculiarities of local climatic conditions are not taken into account.

1. INTRODUCTION

The knowledge of the amount of solar radiation falling on the surface of the earth is of prime importance to engineers and scientists involved in the design of solar energy systems. In particular, many design methods such as *f-chart* (Beckman *et al*., 1977) for thermal systems or *RETScreen* (Leng, 2000; Thevenard *et al*., 2000) for PV systems use monthly average daily radiation on a horizontal surface as an input, in order to predict the energy production of the system on a monthly basis. Monthly average daily radiation on a horizontal surface is summed from daily measurements for many sites in the USA, Canada, Australia, New Zealand, and Europe. However there is a large number of areas, particularly in developing countries, where no measurement is available, or where measurements are available only for limited periods of time. It is therefore of interest, whenever possible, to calculate monthly average daily radiation from other meteorological variables. For both historical and practical reasons the variable most used for that purpose is the number of hours of sunshine per month; this variable is indeed a natural choice, since both radiation and sunshine depend on earth–sun geometry and on the condition of the atmosphere; furthermore sunshine is relatively easy to measure (other estimation methods also use cloud cover or make use of satellite images, but usually require a much greater resolution than monthly data; see for example Davies and McKay (1989)).

The most widely used equation relating radiation to sunshine is the Ångström–Prescott relationship (Black *et al*., 1954; see also Suehrcke (2000) for a historical perspective on the development of this equation):

$$\frac{\overline{H}}{\overline{H}_0} = A + B\frac{n}{N} \tag{1}$$

where $\overline{H}$ is the monthly average daily radiation on a horizontal surface, $\overline{H}_0$ is the monthly mean daily horizontal extraterrestrial radiation, n is the number of hours of bright sunshine per month, and N is the total number of daylight hours in the month. A and B are two constants determined empirically. A and B can assume a wide range of values depending on the location considered; when they cannot be estimated from measured data for a specific location, they can be inferred from correlations established at neighboring locations, an approach that was used for example in Palz and Greif (1996).

The need to rely on an empirical determination of A and B is undoubtedly the greatest shortcoming of the Ångström–Prescott relationship, and it limits the usefulness of the formula. A new approach by Suehrcke (2000) recently established

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a simpler relationship between monthly average daily radiation and sunshine. The relationship is of great interest because it has only one location-dependent parameter (the daily clear† sky clearness index, $\overline{K}_{\text{clear}}$, defined as the ratio of monthly mean daily horizontal surface clear sky radiation to monthly mean daily horizontal extraterrestrial radiation), which is typically between 0.65 and 0.75. Given this small range of values, a reasonable assumption — at least in the absence of additional information — is to set $\overline{K}_{\text{clear}}$ to an average value, say 0.7; in that case Suehrcke's equation becomes universal and can be applied to any location in the world.

In this paper we test Suehrcke's equation using a very large number of measurements, assuming the same value of $\overline{K}_{\text{clear}}$ for all measurements. Given measured values of monthly sunshine, we use Suehrcke's model to compute monthly average daily radiation, then we compare these calculated values to those measured at the same site for the same period of record, and we establish a number of statistical parameters to quantify the mean error and the dispersion of error resulting from the use of Suehrcke's equation. In other words, the question we try to answer is the following: if the monthly amount of sunshine is known for a given site and a given period of record and we use Suehrcke's equation to calculate monthly average daily radiation, what confidence can we have in the result?

2. BACKGROUND: SUEHRCKE'S MODEL

The derivation of Suehrcke's relationship is detailed in Suehrcke (2000) and is briefly summarized here. For any given month with a number n of hours of bright sunshine, the sunshine fraction f_{clear} is defined as:

$$f_{\text{clear}} = \frac{n}{N} \tag{2}$$

where N is the total number of daylight hours in the month. Suehrcke equates this approximately to:

$$\frac{\overline{H}_{\text{b}}}{\overline{H}_{\text{b,clear}}} \tag{3}$$

where $\overline{H}_{\text{b}}$ is the monthly average of daily horizontal surface beam radiation and $\overline{H}_{\text{b,clear}}$ is the monthly average of daily clear sky horizontal surface beam radiation.

To relate $\overline{H}_{\text{b}}$ to the monthly mean daily horizontal surface radiation $\overline{H}$, Suehrcke uses Page's diffuse fraction relationship (Page, 1961):

$$\frac{\overline{H}_{\text{d}}}{\overline{H}} = 1 - c\overline{K} \tag{4}$$

where $\overline{H}_{\text{d}}$ is the monthly mean daily horizontal surface diffuse radiation, c is a constant, and $\overline{K}$ is the monthly mean daily clearness index, defined as:

$$\overline{K} = \frac{\overline{H}}{\overline{H}_0} \tag{5}$$

with $\overline{H}_0$ the monthly mean daily horizontal extraterrestrial radiation. Given that by definition:

$$\overline{H} = \overline{H}_{\text{b}} + \overline{H}_{\text{d}} \tag{6}$$

Eqs. (2) to (5) lead to:

$$\overline{H}_{\text{b}} = c\overline{H}_0\overline{K}^2 \tag{7}$$

The same relationship holds for $\overline{H}_{\text{b,clear}}$:

$$\overline{H}_{\text{b,clear}} = c\overline{H}_0\overline{K}^2_{\text{clear}} \tag{8}$$

where $\overline{K}_{\text{clear}}$ is the monthly average clear sky clearness index, defined as:

$$\overline{K}_{\text{clear}} = \frac{\overline{H}_{\text{clear}}}{\overline{H}_0} \tag{9}$$

where $\overline{H}_{\text{clear}}$ is the monthly mean daily horizontal surface clear sky radiation. Combining Eqs. (7) and (8) leads to the elimination of the constant c and to Suehrcke's relationship:

$$f_{\text{clear}} = \left(\frac{\overline{K}}{\overline{K}_{\text{clear}}}\right)^2 \tag{10}$$

This relationship is particularly elegant, not only because its derivation is so simple, but also because the only semi-empirical constant that is required is $\overline{K}_{\text{clear}}$, the monthly average daily clear sky clearness index. $\overline{K}_{\text{clear}}$ is a measurable quantity; it depends on local atmospheric conditions and, according to Suehrcke (2000), is typically between 0.65 and 0.75.

Although the author suggests that '[this] relationship can be used over the full range of sunshine fraction values from 0.0 to 1.0' (Suehrcke, 2000, top of p. 423) there are two good reasons to doubt this. Firstly, Page's diffuse fraction relationship (Eq. (4); Page, 1961), a key

†In this paper as in Suehrcke (2000) the term 'clear sky' refers to an atmosphere that has no clouds but has all other atmospheric components affecting the atmospheric transmissivity. Other authors also refer to this as the 'cloudless' atmosphere.

element in the derivation, was established from a small number of data and for values of $\overline{K}$ mostly in the range [0.3, 0.7]. There is no indication why this limitation should not also apply to the relationship resulting from the derivation. The second reason is that the derived relationship predicts a clearness index of zero for a sunshine fraction of zero. Since even completely cloudy days provide some solar radiation (as will be confirmed by the data in Fig. 4), this indicates that the relationship is not valid for very low sunshine fractions, although the fact that the slope of the sunshine–radiation relation gets large for low sunshine fractions is probably correct.

3. TEST DATA

To test Suehrcke's relationship we decided to use data collected by the World Radiation Data Center (WRDC, 2000a). This very large data set contains measurements of both global solar radiation and hours of sunshine on a monthly basis. The data set was recently made available on-line (Tsvetkov *et al.*, 1995; WRDC, 2000b), which made the retrieval of data for this study much easier. The data is international in nature, the WRDC acting as a central repository for solar radiation data collected at over 1000 measurement sites throughout the world. The data available for this study covered the period 1964 to 1993. We discarded systematically data flagged as 'missing' or 'doubtful' in the database. We also discarded sites with elevations over 4000 m because too few of those sites were available to be statistically meaningful; measurements prior to 1969 were discarded for the same reason. We kept only those sites and months for which both radiation and sunshine were measured; finally partial years of measurement (fewer than 12 months) were discarded in order to avoid seasonal bias. Months that were partially or totally in polar night were also eliminated. The resultant data set contained 72,984 pairs of monthly radiation and sunshine duration measurements covering a period of 25 years from 1969 to 1993, for a total of 677 different sites.

It should be noted that we had very little control over the quality of the data we used. The data are going through rigorous checks at the WRDC, nevertheless some obvious errors slipped through. A case in point is radiation measurements at S. Tecla and S. Cruz, El Salvador, which are systematically 70% *lower* than measurements at La Union, El Salvador, which is less than 100 km away. Nevertheless such mistakes are expected to be few and not impact on the general conclusions of the study.

It should also be noted that the geographical distribution of the sites we used is somewhat uneven. Fig. 1 shows the location of the sites on a map of the world. Europe contains most sites (46%), followed by Africa (21%), North America (11%), Asia (10%), South America (6%), Oceania (5%), and Antarctica (1%). Finally the number of measurements per site varied from 1 year to 25 years.

The calculation of $\overline{K}$ and f_{clear} from experimental data was done according to Eqs. (2) and (5). The values of N and $\overline{H}_0$ were calculated using formulae for day length and daily extraterrestrial irradiance from Duffie and Beckman (1991) applied to the 'average day of month' for the

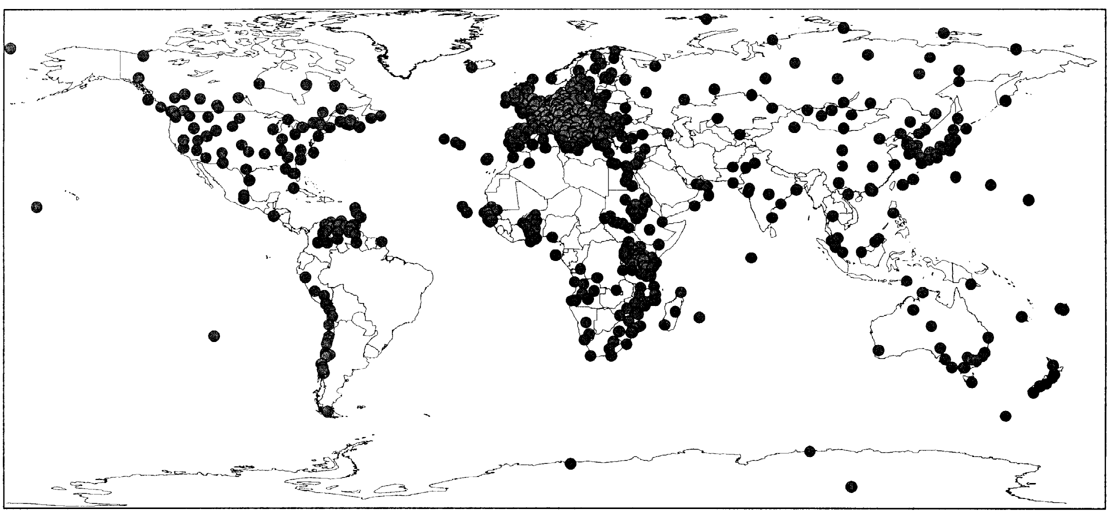

Fig. 1. Location of sites used for the study.

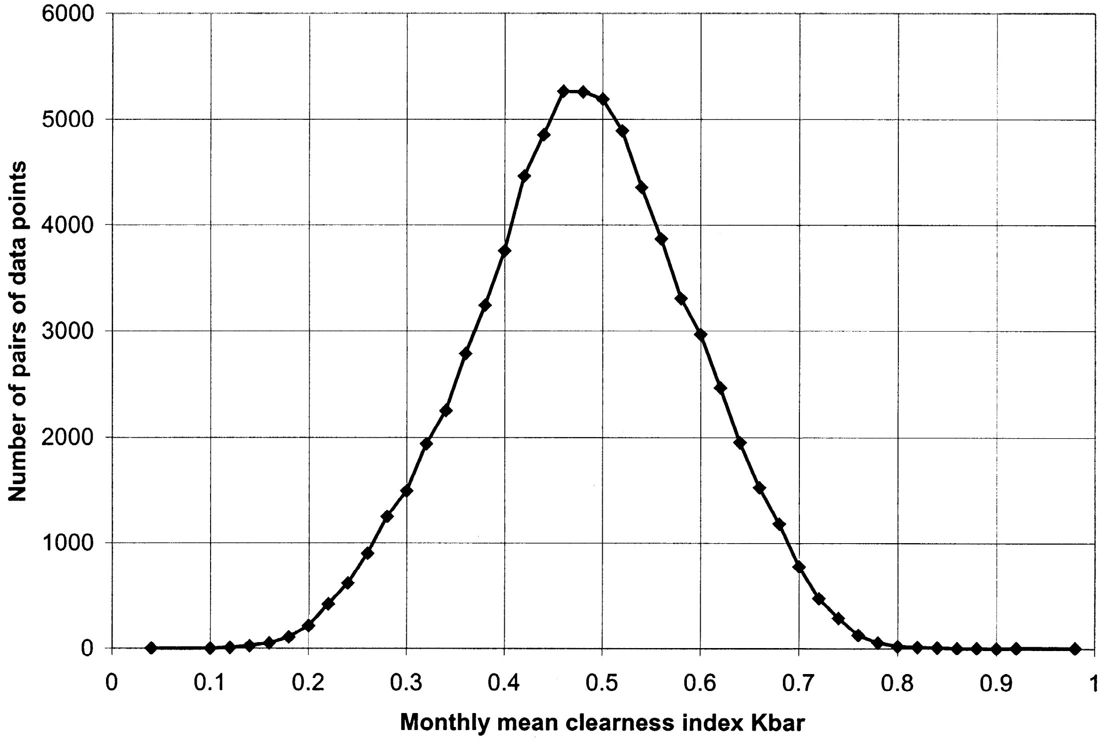

Fig. 2. Distribution of pairs of data points by monthly mean clearness index.

month considered (same reference). We used a value of solar constant equal to 1367 W/m^2.

Test data obtained from the WRDC cover a large range of values for both $\overline{K}$ and f_{clear}. Fig. 2 shows the distribution of $\overline{K}$ (grouped by bins 0.02 wide); the distribution of f_{clear} (also grouped by bins 0.02 wide) is shown in Fig. 3. The $\overline{K}$ distribution curve is close to a bell (Gaussian) shape, whereas the f_{clear} distribution curve follows a triangular shape. Both are centered near 0.5; this suggests that globally, at an average location, the sun will be obscured by clouds approximately half of the day-time; also, the radiation received at the surface will be approximately half of the amount received on a parallel surface outside the atmosphere. Interesting though these observations may be, however, they are of no real consequence for this study.

4. TEST RESULTS

4.1. Test methodology

Suehrcke's formula was used to compute solar radiation on the horizontal for all sunshine measurements in the test data set. Calculated values were then compared to measurements using a variety of methods. In this paper we examine the $\overline{K}$ vs. f_{clear} relationship and the accuracy of the calculation of monthly average daily radiation. $\overline{K}_{\mathrm{clear}}$ was varied over the range 0.65 to 0.75 and the mean bias error of the calculation was found to be very near zero at 0.70; this value was adopted for the rest of the paper.

4.2. $\overline{K}$ vs. f_{clear} relationship

Figs. 4 and 5 show the relationship between $\overline{K}$ and f_{clear}, as predicted by Suehrcke's relationship (Eq. (10)) and as observed experimentally. Fig. 4 shows how the relationship holds for all sunshine, radiation pairs; visually, Suehrcke's equation provides a reasonable fit of the data, and accounts for the inflection observed. In Fig. 5 measured values of f_{clear} were grouped by bins, each 0.02 wide. For each bin, the graph reports the average value of corresponding clearness index observations, as well as the standard deviation and the maximum and minimum values observed in the bin. Eliminating low and high values of f_{clear} (less than 0.1 and more than 0.9, which according to the distribution shown in Fig. 3 represent a small fraction of the data), the relationship proposed by Suehrcke holds, and his curve (Eq. (10)) falls within one standard deviation of experimental values. Fig. 5 also plots the Ångström–Prescott relationship (Eq. (1)) with $A = 0.2336$ and $B = 0.4987$; these two parameters were determined by linear regression (to compensate for the non-

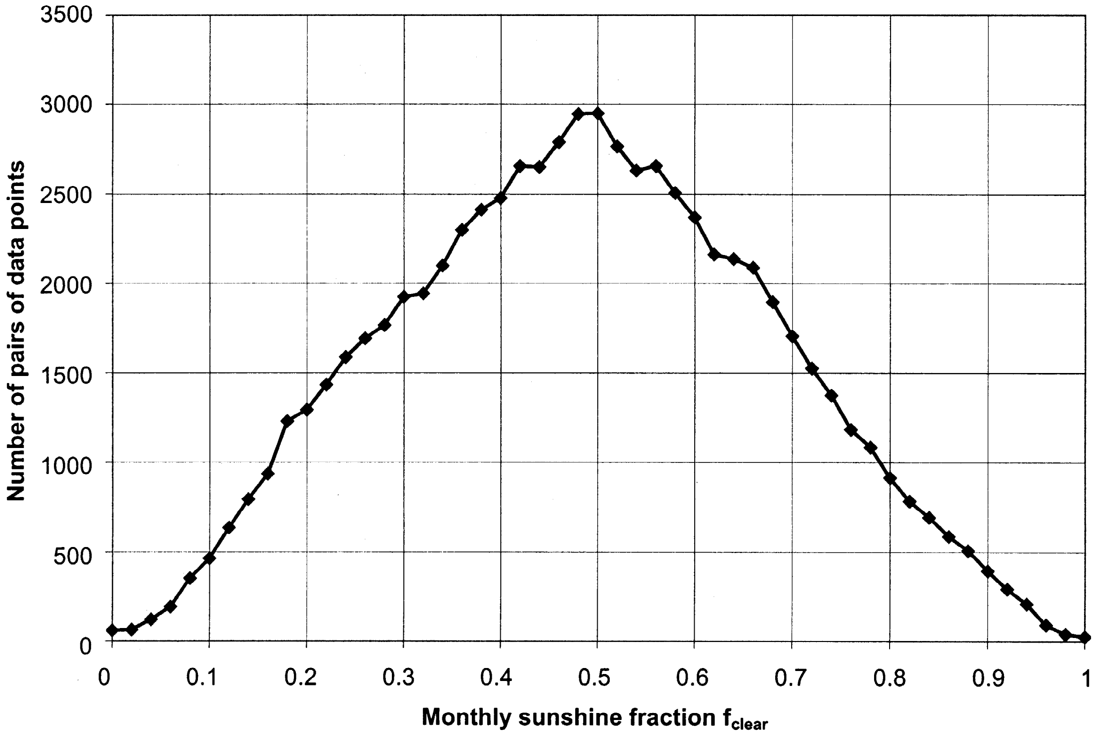

Fig. 3. Distribution of pairs of data points by sunshine fraction f_{clear}.

uniform distribution of the measurements, this regression was done using average values of K for equal intervals of f_{clear} between 0.10 and 0.90). Interestingly enough, there is very little difference between the two models over most of the [0.30, 0.85] range; Ångström–Prescott fits the data

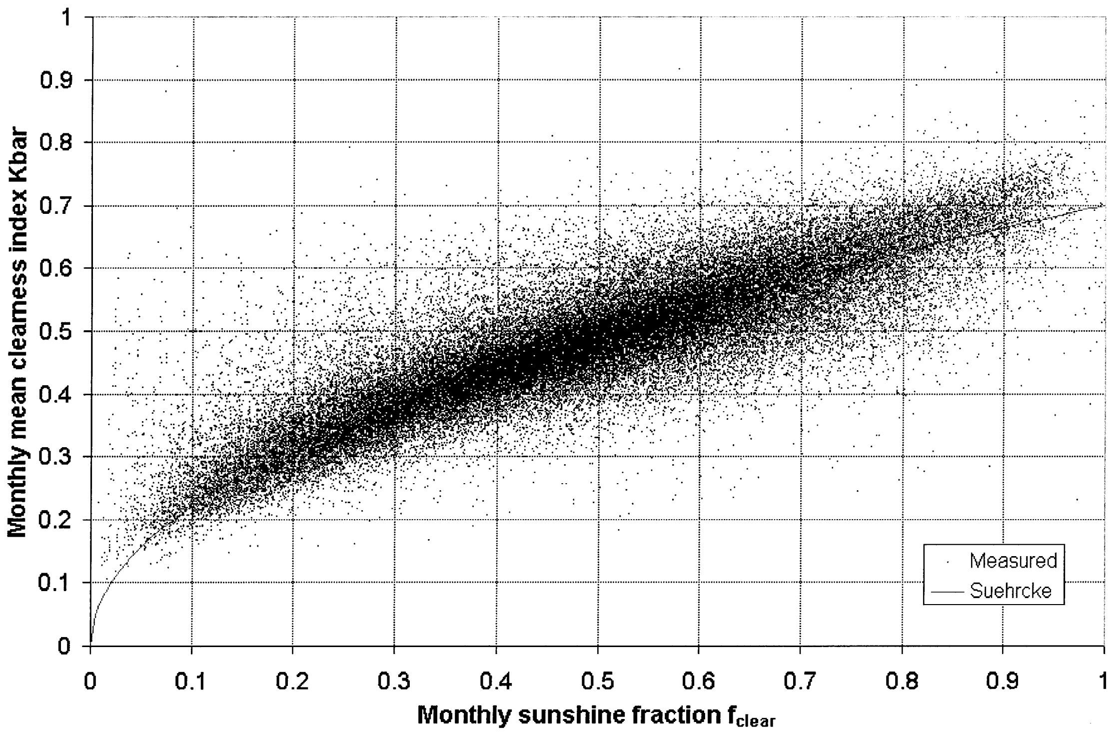

Fig. 4. Experimental verification of Eq. (10) (all pairs of data points).

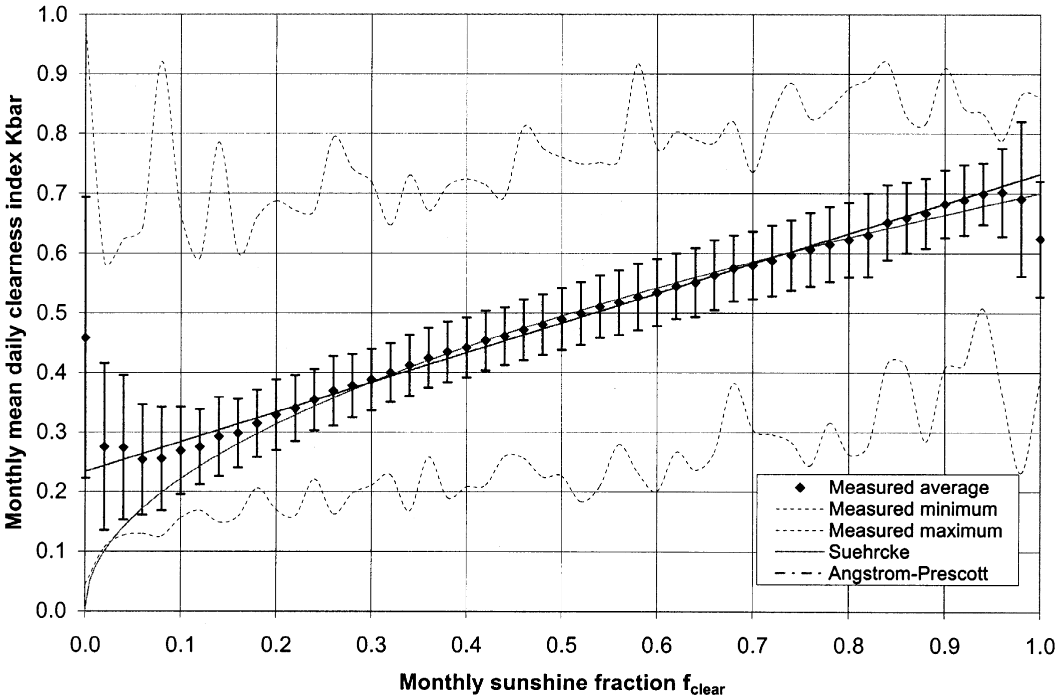

Fig. 5. Experimental verification of Eq. (10) (pairs of data points grouped by bins).

slightly better for f_{clear} greater than 0.85 and less than 0.30 (although neither model claims, or should claim, to be valid below $f_{\mathrm{clear}} = 0.20$; and, as will be noted below, the data may be suspect in that range). But the range of the measured values as indicated by the error bars (standard deviation) and dotted lines (minimum and maximum) shows that while both models are good at predicting the average value for $\overline{K}$, the actual values of $\overline{K}$ show a lot of unaccounted for variation about the average.

At this point it should be noted that for low sunshine fraction ($f_{\mathrm{clear}} < 0.20$) the dispersion of the data in Fig. 4 and in Fig. 5 notably increases. This may point to errors due to sunshine recorder thresholds, particularly with older sunshine recorders. If the threshold is set too high, sunshine at low elevation angles will not be recorded, unlike solar radiation. This can lead to artificially low values of f_{clear} with relatively high corresponding clearness index.

4.3. Calculation of monthly average daily radiation

Figs. 6 and 7 compare monthly average daily radiation calculated with Suehrcke's model (Eq. (10)) with measured values. Fig. 6 compares individual calculated and measured values; in Fig. 7, measured values of $\overline{H}$ were grouped by bins, each 1 MJ/m^2 wide. For each bin, the graph reports the average value of corresponding radiation calculation, as well as the standard deviation and the maximum and minimum values calculated in the bin. The figures show that Suehrcke's model is good at predicting monthly radiation on average, at least when measured radiation is in the range [0, 20] MJ/m^2. However there is a large dispersion of the points and the standard deviation of the prediction is fairly large — around 15% of the average value for each bin. The model also fails to properly calculate higher values of radiation. In the range [20, 30] MJ/m^2, the average values of calculated radiation for each measured radiation bin can be up to 10% too low.

We calculated two statistical indicators in order to answer the question we asked at the beginning of this paper (Section 1). Table 1 contains the mean bias error (MBE), and the root mean square error (RMSE) for radiation calculated by both the Suehrcke and Ångström–Prescott models:

$$\mathrm{MBE} = \frac{1}{p}\sum_{i=1}^{p}\left(\overline{H}_{\mathrm{calc},i} - \overline{H}_{\mathrm{meas},i}\right) \tag{11}$$

$$\mathrm{RMSE} = \left[\frac{1}{p}\sum_{i=1}^{p}\left(\overline{H}_{\mathrm{calc},i} - \overline{H}_{\mathrm{meas},i}\right)^2\right]^{1/2} \tag{12}$$

where p is the number of data points in our test (72,984), the $\overline{H}_{\mathrm{calc},i}$ are calculated values of monthly average daily radiation and $\overline{H}_{\mathrm{meas},i}$ are measured values of the same quantity. Table 1 also expresses these indicators as a percentage of

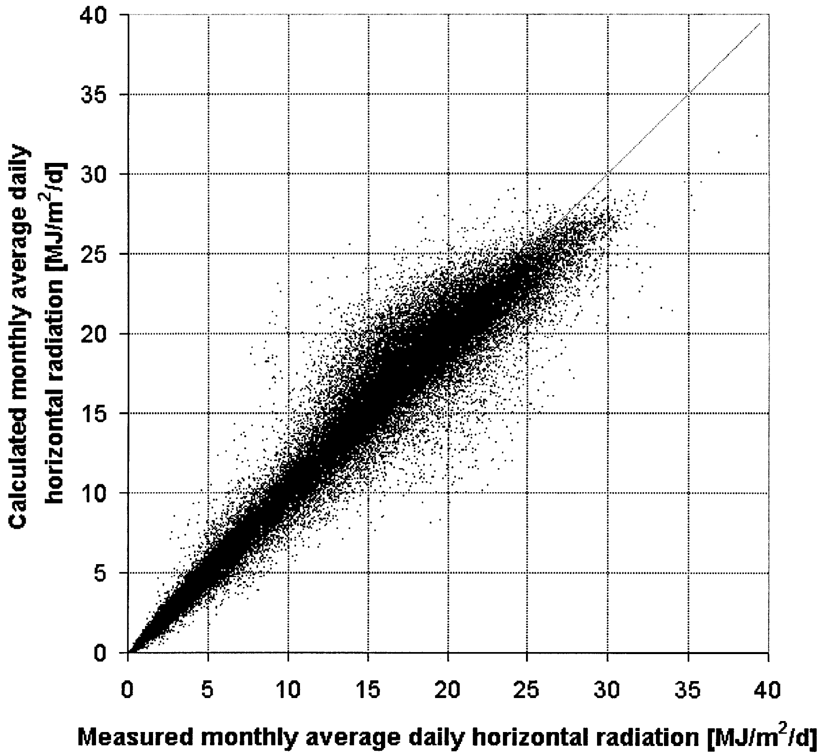

Fig. 6. Comparison of monthly mean daily radiation calculated with Suehrcke's equation to measured values (all points).

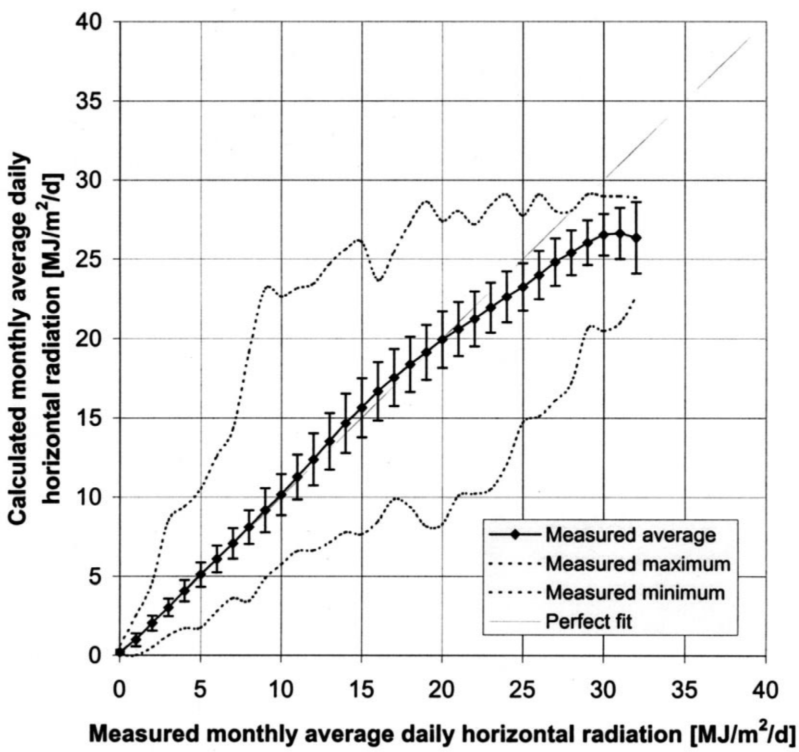

Fig. 7. Comparison of monthly mean daily radiation calculated with Suehrcke's equation to measured values (data points grouped by bins).

Table 1. Statistical parameters for model predictions

Model	MBE $MJ/m^2/day$	MBE %	RMSE $MJ/m^2/day$	RMSE %
Suehrcke	0.0	0.0	1.69	12.2
Ångström–Prescott	0.0	0.0	1.60	11.5

the average measured radiation value for the whole test set. By design in our test, the mean bias error is zero since the values of $\overline{K}_{clear}$ and A and B were chosen to achieve that goal. The RMSE is around 12% of the average radiation for both models, with a slight (but not significant) advantage to the Ångström–Prescott model. Part of that RMSE may be attributable to measurement errors in the values of solar radiation recorded at the WRDC. Nevertheless, this means that a blind application of either model, without supplementary information regarding $\overline{K}_{clear}$ or any local correlation for A and B, will result in estimates of monthly mean solar radiation not accurate by more than 12% on average.

5. DISCUSSION

The key assumption in our study was that $\overline{K}_{clear}$ could be set to a constant value $\overline{K}_{clear} = 0.70$. In reality, $\overline{K}_{clear}$ varies with the type of climate under consideration; Suehrcke mentions the range 0.65 to 0.75 as typical. Sunny and dry climates, for example, may well experience higher values of $\overline{K}_{clear}$ which could account, in part, for some low model predictions when the measured radiation is between 20 and 35 MJ/m^2. It should be noted, however, that monthly radiations calculated with Suehrcke's model are proportional to $\overline{K}_{clear}$, so a variation of this parameter by 0.05 above or below its average value of 0.70 will only increase or decrease the values of radiation calculated by the model by a mere 7%. A much higher variability of $\overline{K}_{clear}$ would be required to account fully for low model predictions.

The simplicity of Suehrcke's formula (Eq. (10)) hinges primarily on the choice of Page's relationship (Eq. (4)) for the calculation of diffuse fraction. We have investigated whether other diffuse fraction relationships, such as the one proposed in Erbs *et al*. (1982), could enhance the accuracy of the model. We found that the use of such formulae provided no significant improvement; furthermore these relationships made Suehrcke's equation much less usable, and in the case of Erbs' formula, turned the equation into an implicit one.

We also investigated the variation of the model's error with other parameters such as latitude, elevation, and monthly average extraterrestrial radiation on a horizontal surface. We found that the error could not be correlated with any one of these parameters alone (we did not test multiple parameters, or combinations). Clearly, both the Ångström–Prescott and the Suehrcke models are close to the limit of prediction accuracy, given the very limited input that they require. The performance of models that calculate monthly average daily radiation simply from the number of hours of bright sunshine per month is apparently limited more by the insufficiency of this sole information than by correctable flaws in the models themselves.

6. CONCLUSIONS

In this paper we have tested Suehrcke's equation for the calculation of monthly average daily radiation on a horizontal surface, given the number of hours of bright sunshine per month. The test was performed with a large number of radiation and sunshine data gathered by the World Radiation Data Center. We also compared the performance of Suehrcke's model with the classical Ångström–Prescott model. We found that Suehrcke's model is adequate at representing, on average, the relationship between monthly clearness index and monthly sunshine fraction, and leads to calculations of monthly daily radiation which, with a proper choice of the clear sky clearness index, match measured values in an average sense. However there is a large dispersion in the predictions of the model and Suehrcke's equation does not prove statistically better than the Ångström–Prescott for the calculation of monthly mean. Furthermore the Suehrcke and Ångström–Prescott relationships between monthly clearness index and monthly sunshine fraction are virtually undistinguishable over most of the range of interest (f_{clear} between 0.3 and 0.8).

Suehrcke's equation remains of prime interest because of its simplicity and the elegance of its derivation. However both the Suehrcke and Ångström–Prescott formulae seem unable to capture, with only sunshine as an input, the breadth of local conditions that can affect the amount of solar radiation at the surface of the earth. It appears that to properly take into account local climatic circumstances, the use of correlations derived from local measurements is still necessary. Such correlations are possible with either model, but they are traditionally used with the Ångström–Prescott formula which provides the advantage of using two empirical parameters, enabling to simultaneously zero the mean bias error and minimize the root mean square error; this procedure, however, is purely empirical and does not benefit from the physical basis of Suehrcke's equation.

NOMENCLATURE

A	first empirical constant of Ångström–Prescott equation (Eq. (1))
B	second empirical constant of Ångström–Prescott equation (Eq. (1))
c	constant in Page's diffuse fraction relationship (Eq. (4))
$\underline{f_{\mathrm{clear}}}$	monthly sunshine fraction
H	monthly average daily radiation on a horizontal surface (MJ m^{-2})
$\overline{H}_{\mathrm{clear}}$	monthly average daily clear sky radiation on a horizontal surface (MJ m^{-2})
$\overline{H}_{\mathrm{d}}$	monthly mean daily horizontal surface diffuse radiation (MJ m^{-2})
$\overline{H}_{\mathrm{b}}$	monthly average of daily horizontal surface beam radiation (MJ m^{-2})
$\overline{H}_{\mathrm{b,clear}}$	monthly average of daily clear sky horizontal surface beam radiation (MJ m^{-2})
$\overline{H}_{0}$	monthly mean daily horizontal extraterrestrial radiation (MJ m^{-2})
$\overline{K}$	monthly mean daily clearness index
$\overline{K}_{\mathrm{clear}}$	monthly average clear sky clearness index
n	number of hours of bright sunshine per month (h)
N	total number of daylight hours per month (h)

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